Collapsing Gas Clouds - Stability

Gas clouds in interstellar space are acted upon by external pressure and their own gravity, and would otherwise collapse, but if they are hot enough, they can remain stable for a long time. That seems to be the case for objects called Bok Globules.

This photo of Thackeray's Globule (IC-2944) taken by the Hubble Space Telescope may be a stable dark cloud containing 10 times the mass of our sun at a temperature of less than 100 K.

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P so that

$$\boxed{P = \frac{3MkT}{4\pi R^3 \mu m} - \frac{3GM^2}{20\pi R^4}}$$

where k, G and μ are constants.

Problem 1 - At what critical radius will the cloud start to collapse for a given mass and temperature?

Problem 2 - What will be the critical external pressure at this radius?

Answer Key

Problem 1 - The problem states that the mass and temperature are held constant, so the only free variable is R. For complicated equations, it is always a good idea to group all constants together and define new constants. You can later replace the new constants by the old ones. Let's define A = ($3MkT/4\pi\mu m$) and B=($3GM^2/20\pi$), then the equation becomes $P = AR^{-3} - BR^{-4}$. To find the extremum, we calculate dP/dR and set this equal to zero. This gives us $dP/dR = A(-3)R^{-4} - B(-4)R^{-5} = 0$. This leads to R = 4 B/3 A which upon substituting back for the definitions of A and B gives us

$$R_C = (12/45)\, GM\, \mu\, m/(k\, T)$$

Problem 2 - To find the critical pressure, simply substitute R_c for R in the original equation for P. This algebra is a bit messy, so be careful of the many factors.

$$P = \left[\frac{3MkT}{4\pi\mu m} - \frac{3GM^2}{20\pi}\left(\frac{12}{45}\,\frac{GM\mu m}{kT}\right)^{-1}\right]\left(\frac{12}{45}\,\frac{GM\mu m}{kT}\right)^{-3}$$

$$P = \left[\frac{3MkT}{4\pi\mu m} - \frac{3MkT}{4\pi\mu m}\left(\frac{9}{12}\right)\right]\left(\frac{12}{45}\,\frac{GM\mu m}{kT}\right)^{-3}$$

$$P = \frac{3\ \ 45^3}{12\ \ 12^3}\left[\frac{3MkT}{4\pi\mu m}\right]\left(\frac{kT}{GM\mu m}\right)^3$$

$$P = \frac{10125}{1024}\left(\frac{kT}{\mu m}\right)^2\frac{1}{G^3M^2}$$